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# Teaching and Learning using Mathematics Software "The New Challenge"

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**Abstract**

Teaching and learning in mathematics curriculums in Universities by using mathematical software can be a difficult and demanding task, especially for novice learners. This paper presents efficient mathematical tool for teaching and learning of Linear Algebra courses. MAPLE software tool was used for teaching and learning of parts of the Linear Algebra course. Using MAPLE in teaching and learning mathematical concepts is a great challenge both from a didactical and a scientific point of view. In this paper we provide mathematical examples by using mathematical software and necessary steps as evidences that didactically it increases mathematical skills. By using interactive Maple worksheets and animated graphics, students can find the opportunity of numerous experiments that provide well understanding. Further the use of Maple provide conceptual and meaningful understanding for the student, several Maplet can be designed to see, geometrical application of Linear Algebra topics. Indeed, utilizing ICT and particular the use of interactive facilities of Maple in teaching and learning which will provide a new challenge to both mathematics educators as well as students.


***Keywords:*** Teaching and learning; MAPLE; Mathematics software; utilizing ICT

## 1. Introduction

Today, while education advances then increasing number of teachers and lecturers are incorporating technology into their classroom and a larger number of educationist willing to do so. It is also well known that the use of technology in the classroom makes higher-level mathematical activities accessible to students. Thus the technology can strengthen students learning process by presenting content numerically, graphically, as well as symbolically without extra burden of spending time to calculate the complex computational problems by hand. Instead of focusing on the computations, technology can help to foster students and gain the ability and skils to make connections between the concepts during the solution and proving process. Thus the use of technology in the classroom can lead to advance in conceptualization. In addition, integrating technology into a mathematical courses will increase awareness among the students and help them to evaluate and correct themselves. The technology also helps students to make connections within mathematics and beyond, by making the learning process more realistic in context (Horton, Storm, and Leonard, 2004). Further the integration of ICT can also contribute significantly to student engagements, motivation, as well as attitude toward mathematical courses.

However technology should not be the sole venue to present the content rather the ICT is one of many tools to help students to learn materials in a deeper and more meaningful way. Moreover, the way in which technology is used dictates the effectiveness of it thus should never been considered to replace the activities that connect to content. Instead, the main aim in use of technology should be to enrich students' learning process by providing interactive experience.

Today there are several popular technological tools that are widely used in mathematics education. A number of symbolic packages for mathematics courses are available such as MATHEMATICA, MATLAB or MAPLE for education as well as research. Some of the these symbolic packages enable the students to achieve high level of logical-analytical reasoning by visually

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supporting the concepts and further to view the proofs with graphical expressions see for example, [4, 5, 14]. Further, many researchers use the CAS to measure and evaluate the student progress in learning of mathematical concepts by generating the dynamic visualizations [Hayden, Heid, Kendal and Stacey et. al]. Similarly, the Maple has being used to teach advanced calculus such as double integration [Mathews, 1990], number theory [Cheung, 1996], complex graph sketching [Kong and Kwok], as well as mathematical analysis in applied sciences such as electronic signals and circuits [Royrvik, 2002]. Symbolic packages are also used in teaching other subjects such as physics and engineering [Johnson and Buege, 1995; Ward, 2003] and computation [Tonkes et. al, 2005]. In the CAS the students are provided with opportunities to perform mathematics exploration and problem solving.

Based on a powerful scientific programming environments and packages of special routines in their built in libraries, Maple is one of the most popular systems since it is well-suited to aid college students to learn mathematics through verifying calculation and plotting complicated graphs, and also combines mathematical capabilities with a text editor. In this paper, we develop an efficient teaching-learning tool to aid in teaching and learning mathematical concepts of linear algebra courses. The interactive CAS tools support the student to understand linear algebra and gain some skills. Thus the objectives of this study as follows:

(i) To receive an instant feedback by using the CAS when trying out ideas and encourage the students to use conjecture as well as exploration.
(ii) Using the mathematical software to carry out the necessary computational calculations and to draw the students' attention to focus on strategies.
(iii) To provide collaboration among the students in the completion of their assignment and projects.

## 2. Preliminaries and overview

The Maple provides over 45 interactive online tutorials to help in the learning of several topics that are covered in the university level such as Calculus, Multivariate Calculus, Vector Calculus, Differential Equations, Linear Algebra, Complex Variables, and Numerical Analysis. Some of the tutorials are also accessible by using the student packages that is the collection of subpackages designed to assist the standard undergraduate mathematics. The subpackages also contain many commands with displaying functions, computations, and theorems in various ways by stepping through important computations.

i. The visualization commands are tools that create plots and animations showing the geometric interpretation of important concepts.
ii. The computation commands help to study the techniques of computation while solving problems. For example, using the Calculus1 package, you can differentiate a function one step at a time by specifying the needed differentiation rule applied at each step. At any time during a single-step computation, you can request a hint about the next step, which you can then apply to the problem, or you can proceed directly to the final answer.
iii. The interactive commands help to explore concepts and solve problems using a point-and-click interface. These commands launch tutors that provide a graphical interface to visualization and computation commands described above.

## 3. Interactive Tutors of Linear algebra in Maple

In this study we only focus on how Tutorials in Maple can be used some topics in Linear algebra course that contain over 9 interactive tutorials, such as plotting eigenvector, computation of eigenvalues, solving the linear system. In fact Maple can be used in many courses since Tutorials are interactive tools that aid in the learning of precalculus, calculus, multivariate calculus, vector calculus, and differential equations to understand the concepts. These tutorials are easily accessible by selecting Tools + Tutors. See figure (1).

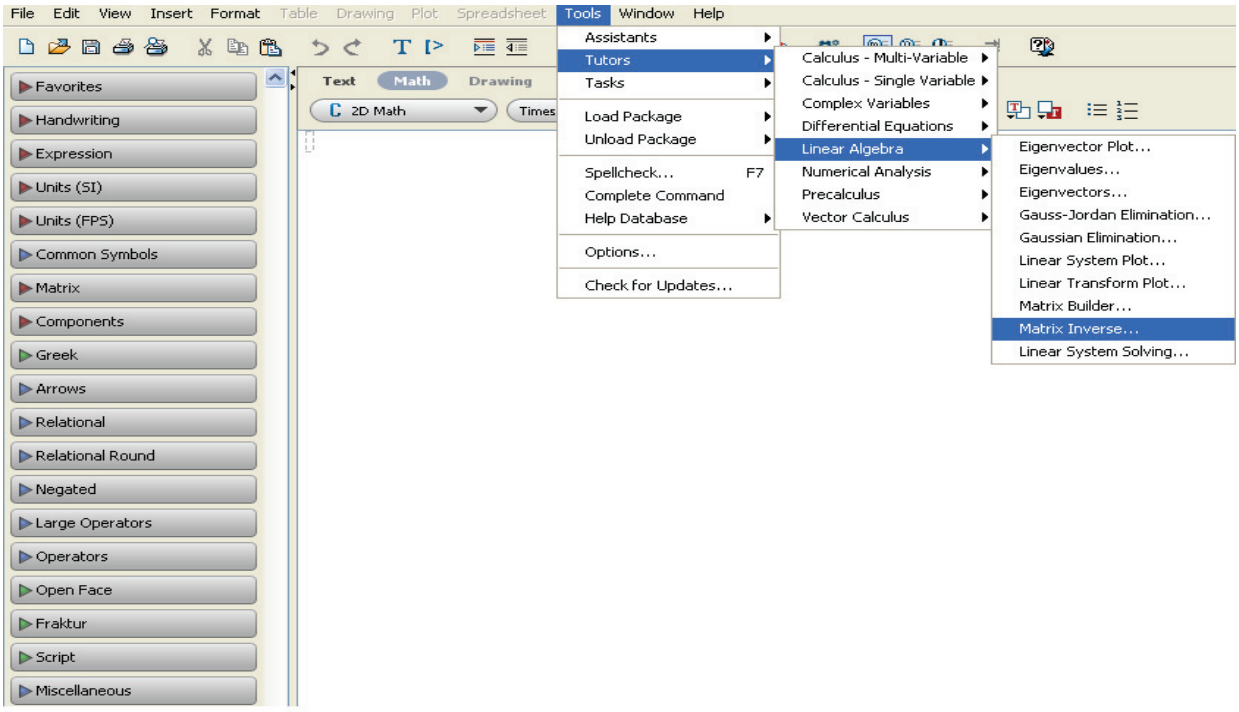


Figure 1: Accessing Tutors from the Tools Menu

Here we include some examples of using Maple to see solutions step by step of problems in linear algebra (matrix inverse). The following example shows the inverse of a matrix without any explanation, and the students can use Maple to check solution which obtained by hand.

**Example 3.1.** *If we have a matrix having size* 3 *x* 3 *and want to explain the students how we can find inverse of this matrix by using Maple software, first we may find matrix directly by one step or if we use Linear algebra package as follow*

```
> With(LinearAlgebra):
> A:=any square matrix;
> MatrixInverse(A);
```

But if we want to show the student the necessary steps in finding the inverse; one of the way is to use the idea of augmented matrix that is setting the extended matrix [A : I] and try to convert into [I : $A^{-1}$], we will explain by next example and figures 2.

**Example 3.2.** *Let M any square matrix*

```
> with(Student [LinearAlgebra]):
> M := Insert matrix ;
> InverseTutor(M);
```

Here, if the lecturer wants to explain the solution step-by-step then will press on the icon ``Next Step" which is appeared as in the figure 2.

For example, in the following example we provide the steps of solution in order to find eigen values and eigen vectors, without extra any powerpoint slides we can use only CAS Maple to explain for student Eigenvectors subject step by step as follow:

Suppose that we have a 3 *x* 3 matrix, if we select Tools → Tutors → Linear Algebra → Eigenvectors then we can have the interactive worksheet as follows:

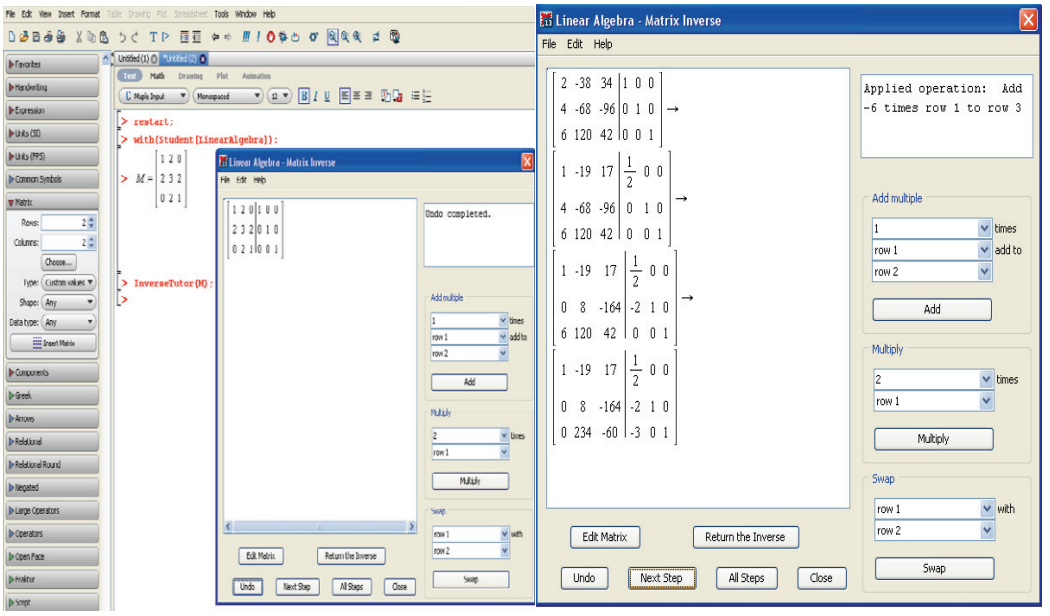


Figure 1: To find matrix inverse step by step

Or we can write simple program as follow:

**> restart;**

**> Student [LinearAlgebra] [EigenvectorsTutor] ();**

After running we will see the following steps one after other

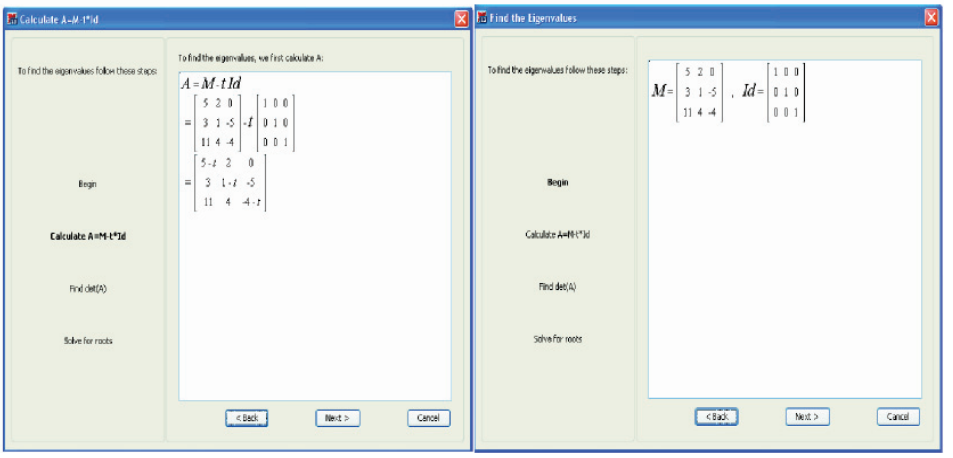


Figure 2: To find eigenvalue

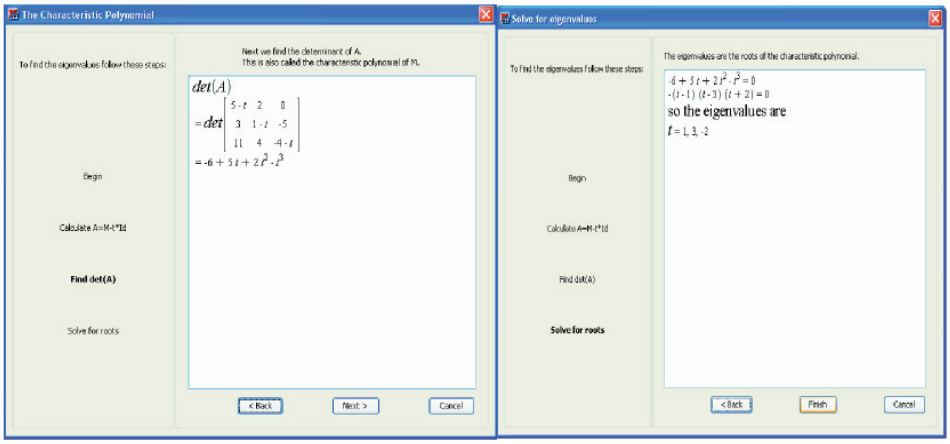


Figure 3: To find the eigenvalues

Now if we want to continue we just click next step and see the details of finding eigenvector.

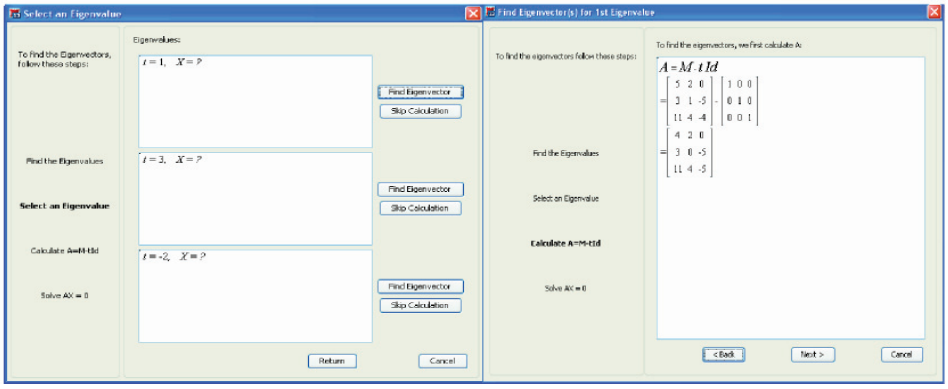


Figure 4: To find eigenvectors

In fact there are several tutorial classes and some of the tutorials are also accessible through the Student package. For example, the tutorials on Differential Equations, DE Plots, is also accessible through the DEtools package. In fact the Student package is a collection of sub packages that are designed to assist with the teaching and learning of standard undergraduate mathematics.

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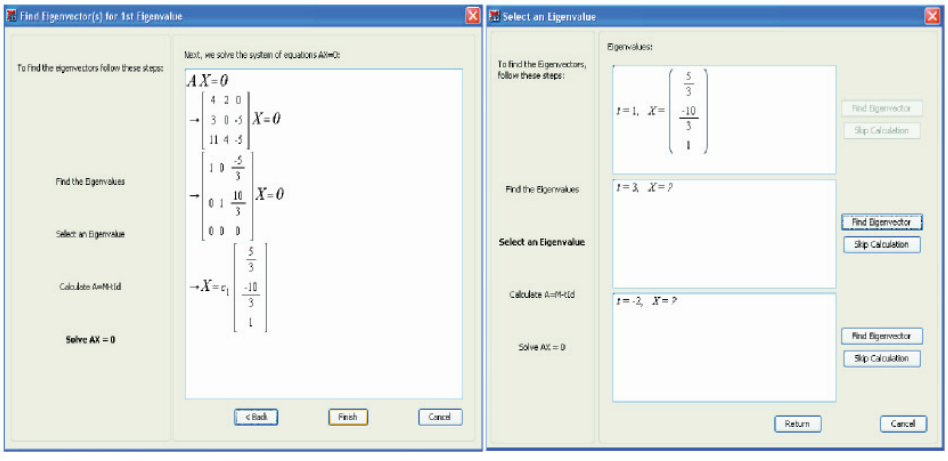


Figure 5: To find eigenvector

The sub packages contain many commands for displaying functions, computations, and theorems in various ways, and support for stepping through important computations. The visualization commands are executive tools that produce several types of plots and animations in order to show the geometric interpretation of important concepts. Similarly, the computational commands help the student to study the techniques of computation while solving the problems. For example, using the Calculus 1 package, we can differentiate a function one step at a time by specifying the way of differentiation rule that applicable at each step. At any time during a single-step computation, we might request a hint about the next step, then apply to the problem, or directly to the final answer. The interactive commands help us explore the concepts and to solve problems by using a point-and-click interface. These commands are available in tutorials that provide a graphical interface to visualization and computation commands. See Figure 7 for an example for one of the interactive commands.

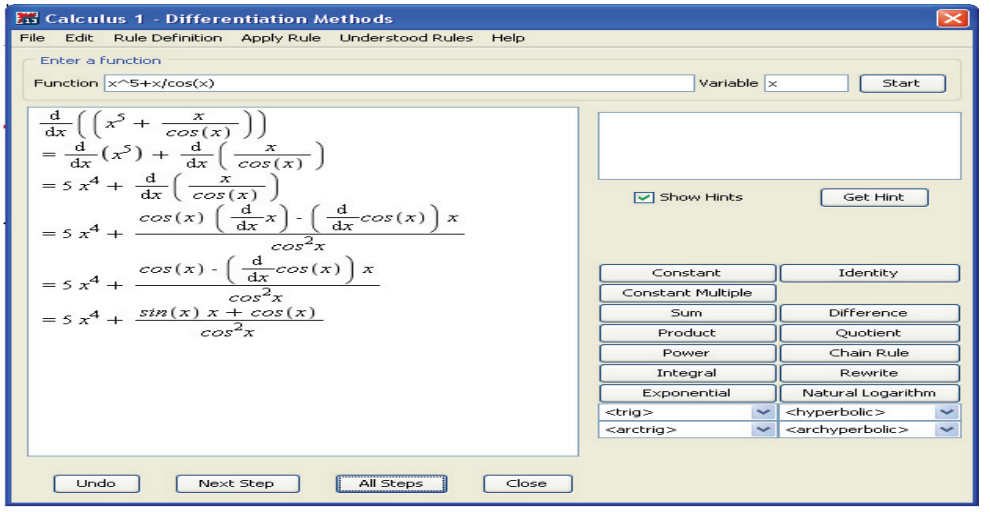


Figure 6: Calculus - Single Variable, Differentiation Methods Tutor

## 4. Conclusion

The Computer algebra systems (CAS's) such as MAPLE, MATLAB, MUPAD and MATHEMATICA can be used as a powerful assistants to perform the symbol manipulations and computations in algebra as well as calculus. It has been suggested that these systems will benefit undergraduates and postgraduates in mathematics, engineering and physics by keeping track of the details in complicated manipulations. This work only shows how the symbolic linear algebra MAPLE can be used to help the understanding of both the theoretical and computational aspects of some topics in Linear Algebra that is available on the computers in Math department as well as in INSPEM.

The eigenvalues and eigenvectors are familiar topics for the second year mathematics and statistics students. It is considered as the foundation of much theory in further courses such as Real Analysis and Applied Mathematical Courses. However, it is usually viewed by students to be difficult to understand. Using assistance from MAPLE to construct the eigenvectors and eigenvalues of Matrices, and examine and manipulate further on the matrices, thus gives the students more time to focus the attention on the important properties and principles. Thus the use of the MAPLE will generate enthusiasm for both students and lecturers.

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